Find resistor values for arbitrary programmable-amplifier gains

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When available fixed-gain values match design requirements, a PGA (programmable-gain-amplifier) IC offers a drop-in choice, but what does a designer do when a suitable PGA is unavailable? Before the PGA's advent, a circuit designer who needed selectable, fixed amounts of gain chose a suitable operational amplifier and designed a switched-resistor gain-setting network. This Design Idea discusses two methods of designing the desired resistive network.

Figure 1 shows a series-ladder-resistor network comprising a string of resistors whose junctions connect to switch-selectable taps that determine the circuit's gain. Little current flows through the switch, and the resistance of the switch thus doesn't affect the design. A circuit with N discrete-gain values requires an N-position switch, usually an analog multiplexer, and N+1 resistors in its ladder. **Equation 1** describes the circuit's gain in the general case:

$$\mathrm{GAIN}[n] = \frac{\sum_{i=1}^{n} R_i}{\sum_{i=n+1}^{N+1} R_i} + 1. \qquad (1)$$

You can solve **Equation 1** for the resistor summations and expand a few terms as follows:

$$\frac{\sum_{i=1}^{n} R_i}{\sum_{i=n+1}^{N+1} R_i} = \mathrm{GAIN}[n]-1. \qquad (2)$$

$$\sum_{i=1}^{n} R_i = (\mathrm{GAIN}[n]-1) \times \sum_{i=n+1}^{N+1} R_i. \qquad (3)$$

$$R_1 = (\mathrm{GAIN}[1]-1) \times (R_2 + R_3 + \ldots + R_{N+1}), \qquad (4)$$

$$R_1 + R_2 = (\mathrm{GAIN}[2]-1) \times (R_3 + \ldots + R_{N+1}), \qquad (5)$$

and

$$R_1 + R_2 + R_3 + \ldots + R_N = (\mathrm{GAIN}[N]-1) \times (R_{N+1}). \qquad (6)$$

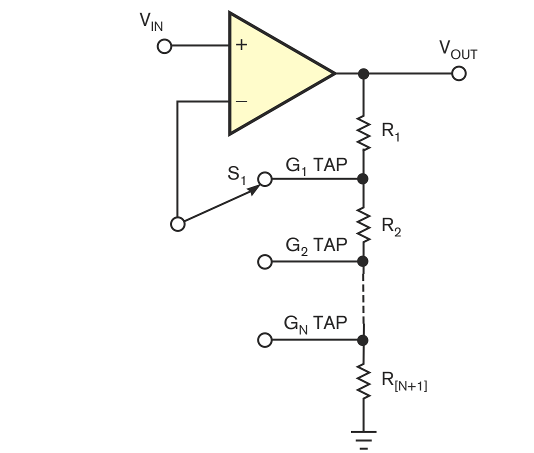

Figure 1 A series-resistor-ladder network and a single-pole, multiple-throw switch form a custom-value programmable-gain amplifier.

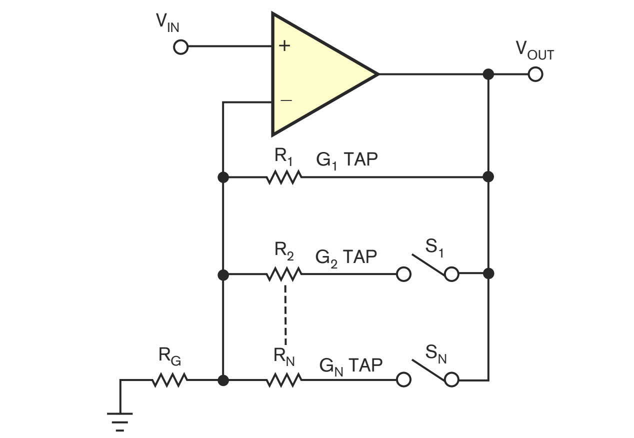

Figure 2 In a parallel-resistor-ladder network, connecting one resistor at a time in parallel with R_1 determines the circuit's gain.

Next, normalize R_1 to 1Ω and solve the equations for R_1:

$$1=(\text{GAIN}[1]-1)\times(R_2+R_3+\ldots+R_{N+1}), \quad (7)$$

$$1=-R_2+(\text{GAIN}[2]-1)\times(R_3+\ldots+R_{N+1}), \quad (8)$$

and

$$1=-R_2-R_3-\ldots-R_N+(\text{GAIN}[N]-1)\times(\ldots+R_{N+1}). \quad (9)$$

$$\begin{bmatrix} \text{GAIN}[1]-1 & \text{GAIN}[1]-1 & \text{GAIN}[1]-1 & \text{GAIN}[1]-1 \\ -1 & \text{GAIN}[2]-1 & \text{GAIN}[2]-1 & \text{GAIN}[2]-1 \\ \ldots & \ldots & \ldots & \ldots \\ -1 & -1 & -1 & \text{GAIN}[N]-1 \end{bmatrix} \times \begin{bmatrix} R_2 \\ R_3 \\ \ldots \\ R_{N+1} \end{bmatrix} = 1. \quad (10)$$

A network that synthesizes N gain values results in an N×N matrix whose upper echelon equals the desired gains minus one, in ascending order, and its lower echelon equals negative one. To produce the resistor values for the desired gains, invert the matrix and calculate its dot product with a unity matrix. For example, a circuit requiring four gain values of three, five, 24, and 50 also requires five resistors. Stuffing and solving the matrix yields:

$$\begin{bmatrix} 2 & 2 & 2 & 2 \\ -1 & 4 & 4 & 4 \\ -1 & -1 & 23 & 23 \\ -1 & -1 & -1 & 49 \end{bmatrix} \times \begin{bmatrix} R_2 \\ R_3 \\ R_4 \\ R_5 \end{bmatrix} = 1. \quad (11)$$

$$\begin{bmatrix} R_2 \\ R_3 \\ R_4 \\ R_5 \end{bmatrix} = \begin{bmatrix} 0.2000 \\ 0.2375 \\ 0.0325 \\ 0.0300 \end{bmatrix}. \quad (12)$$

Scale the resistors' values to 1 kΩ and select the closest available standard resistor values to produce gains of:

$$\begin{bmatrix} R_2 \\ R_3 \\ R_4 \\ R_5 \end{bmatrix} = \begin{bmatrix} 200 \\ 237 \\ 32.4 \\ 30.1 \end{bmatrix}. \quad R_1=1\text{ k}\Omega. \quad \text{GAINS}=\begin{bmatrix} 3.002 \\ 5.007 \\ 23.99 \\ 49.82 \end{bmatrix}. \quad (13)$$

Figure 2 shows a parallel-resistor-ladder network. To select a gain value, connect an additional resistor in parallel with the other resistors. A circuit with N discrete gains requires N resistors in the ladder; an additional gain resistor, R_G; and N−1 switches. **Equation 14** describes the circuit's gain in the general case:

$$\text{GAIN}[n]=\frac{R_1\|R_2\|\ldots\|R_N}{R_G}+1, \quad (14)$$

and **Equation 15** describes the parallel-resistor combination for each gain:

$$R_P[n]=(\text{GAIN}[n]-1)\times R_G. \quad (15)$$

The nth value of R_P equals the nth−1 value of R_P in parallel with the ladder's nth resistor. Solve the following equations for the nth resistor value:

$$R_P[n]=R_P[n-1]\,\|\,R_n, \quad (16)$$

$$R_1=(\text{GAIN}[1]-1)\times R_G, \quad (17)$$

and

$$R_n=\frac{R_P[n]\times R_P[n-1]}{R_P[n-1]-R_P[n]}. \quad (18)$$

To find the desired network's resistors, select the desired gain values and R_G and then use **Equation 14** to calculate the parallel values. Use the resulting values to solve **Equation 15** and find the required resistor values. As in the previous example, a circuit must produce gain values of three, five, 24, and 50. Four gain values require four resistors. Let $R_G=1\Omega$. Solving **Equation 14** for the parallel-values matrix yields:

$$\begin{bmatrix} R_P[1] \\ R_P[2] \\ R_P[3] \\ R_P[4] \end{bmatrix} = \begin{bmatrix} 49 \\ 23 \\ 4 \\ 2 \end{bmatrix}. \quad (19)$$

Substituting these values into **Equation 15** yields the resistors' values:

$$R_1=49\times1=49\Omega, \quad (20)$$

$$R_2=\frac{23\times49}{49-23}=43.35\Omega, \quad (21)$$

$$R_3=\frac{4\times23}{23-4}=4.842\Omega, \quad (22)$$

and

$$R_4=\frac{2\times4}{4-2}=4\Omega. \quad (23)$$

Scaling to 1 kΩ and selecting the closest available standard-value resistors yields gains of:

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} 48,700 \\ 43,200 \\ 4870 \\ 4002 \end{bmatrix}. \quad R_G=1\text{ k}\Omega. \quad \text{GAINS}=\begin{bmatrix} 49.7 \\ 23.9 \\ 5.02 \\ 3 \end{bmatrix}. \quad (24)$$

Reference 1 provides a review of the matrix math. **EDN**

REFERENCE

1. Freeman, Larry, "Review of Matrices," Math Refresher, Dec 19, 2005, http://mathrefresher.blogspot.com/2005/12/review-of-matrices.html.