Calculating resistances for sum and difference networks

by D. Sheingold
Analog Devices Inc., Norwood, Mass.

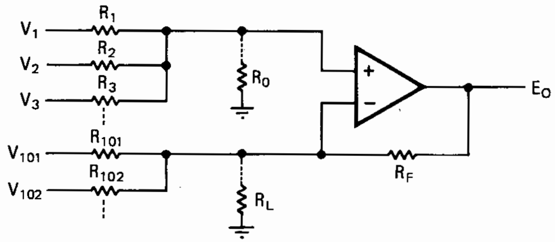
Whenever signals must be added and/or subtracted, a few simple computations will yield resistance values that provide equal resistive loading at the two inputs of an operational amplifier to minimize offset-current errors. The loading resistance can have any desired value.

Figure 1 shows the general sum or difference network; it produces an output voltage given by

$$E_O = a_1V_1 + a_2V_2 + \ldots$$
$$-(b_1V_{101} + b_2V_{102} + \ldots)$$

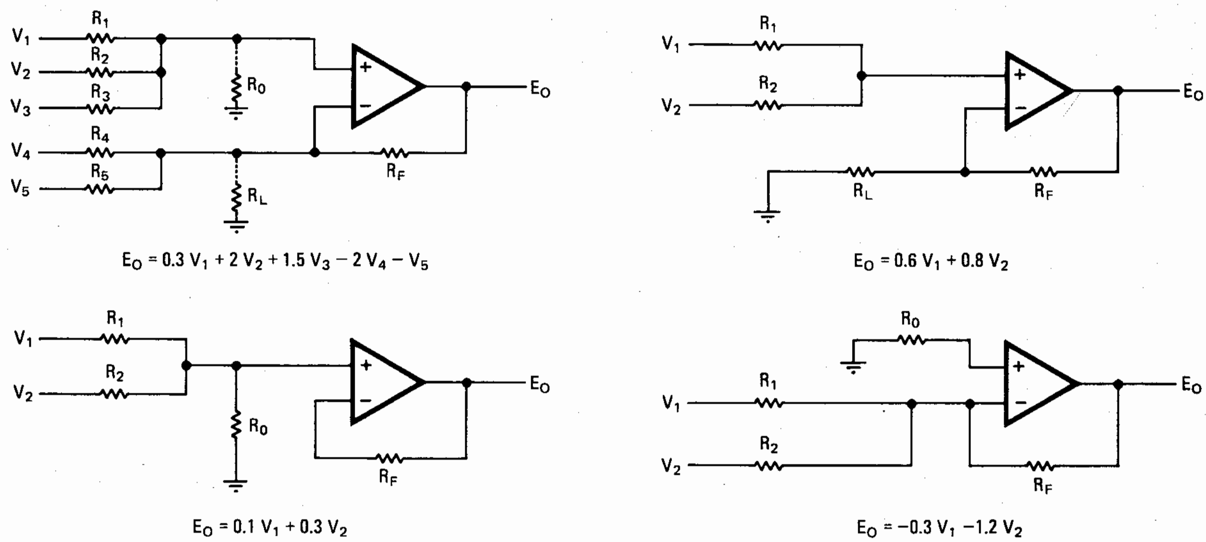
where the Vs are input voltages. The voltages that are to be added (V_1, V_2, V_3 . . .) are applied to the noninverting terminal of the operational amplifier through resistors R_1, R_2, . . . , and the voltages that are to be subtracted (V_{101}, V_{102}, . . .) are applied to the inverting terminal through resistors R_{101}, R_{102}, ... Shunt resistor R_0 or R_L and feedback resistor R_F complete the network. The values of all the resistors are found by these simple rules:

- Decide what composite load resistance, R_p, should be presented to the input terminals of the op amp. A value of 5 kilohms for R_p provides good bandwidth and low noise pickup without too much loading of the input sources or the output.
- Add up the positive coefficients (call this sum Σa).
- Add up the negative coefficients (call this sum Σb), and add 1.00.
- If Σa is greater than $(1+\Sigma b)$, the network must include an R_L (for gain). If Σa is less than $(1+\Sigma b)$, the network must include an R_0 (for attenuation). If Σa is equal to $(1+\Sigma b)$, neither R_L nor R_0 is used.
- Find R_F by taking the larger of Σa or $(1+\Sigma b)$, and multiplying it by R_p. (The number that multiplies R_p here is called the closed-loop gain or "noise gain.")



$$E_O = a_1V_1 + a_2V_2 + a_3V_3 + \ldots - (b_1V_{101} + b_2V_{102} + \ldots)$$

1. Summing circuit. Output voltage from operational amplifier is sum of positive and negative terms that are related to input voltages by positive or negative coefficients. Signs of terms depend on which input terminal is fed, and magnitudes of terms depend on voltages and resistances. Simple procedure determines resistance values that yield the desired output while making op-amp input terminals see equal resistive loadings of any desired level. Circuit may include balancing resistor R_0 or R_L or neither, but never requires both.



2. Sample problems. Examples in text refer to these circuits. Resistor values are calculated on basis of 5-kilohm loading, a value chosen for convenience, at each input terminal of op amp. The circuit in (a) is the most general adder-subtractor; (b) and (c) are simple adders; and (d) is an inverting adder. Each example highlights a particular feature of the calculation procedure.

■ R_L or R_O is equal to R_F divided by the absolute value of $(1+\Sigma b-\Sigma a)$.

■ The value of each of the other resistances is found by dividing R_F by the associated coefficient: i.e. $R_1 = R_F/a_1$, $R_{102} = R_F/b_{102}$, and so forth.

As an example, the resistors for the network in Fig. 2(a) can be found by following the above rules:

Choose $R_p = 5\ k\Omega$
$\Sigma a = 3.8$
$(1 + \Sigma b) = 4.0$
$(1+\Sigma b) - \Sigma a = 0.2$ *(An R_O is needed.)*
$R_F = 4 \times 5\ k\Omega = 20\ k\Omega$ *(Closed-loop gain is 4.)*
$R_O = 20/0.2 = 100\ k\Omega$
$R_1 = 20/0.3 = 66.7\ k\Omega$
$R_2 = 20/2 = 10\ k\Omega$
$R_3 = 20/1.5 = 13.3\ k\Omega$
$R_4 = 20/2 = 10\ k\Omega$
$R_5 = 20/1 = 20\ k\Omega$

As a check, the parallel combination of R_1, R_2, R_3, and R_O is 5 kΩ, and parallel combination R_4, R_5, and R_F is also 5 kΩ. (There is no R_L in the network.) The gains for V_4 and V_5 are $-20/10 = -2$, and $-20/20 = -1$, respectively. The gain for V_1 is the product of noise gain and attenuation (in the voltage divider that consists of R_1 and the parallel combination of R_2, R_3, and R_O); this product is $4 \times 0.075 = 0.3$. The gain for V_2 is $4 \times 0.5 = 2$, and the gain for V_3 is $4 \times 0.375 = 1.5$.

A second example is the summing circuit in Fig. 2(b).

Again choose $R_p = 5\ k\Omega$
$\Sigma a = 1.4$
$(1+\Sigma b) = 1 + 0 = 1.0$
$\Sigma a - (1+\Sigma b) = 0.4$ *(An R_L is needed.)*
$R_F = 1.4 \times 5\ k\Omega = 7\ k\Omega$ *(Noise gain is 1.4).*
$R_L = 7/0.4 = 17.5\ k\Omega$
$R_1 = 7/0.6 = 11.7\ k\Omega$
$R_2 = 7/0.8 = 8.8\ k\Omega$

A check of these results shows that both input terminals are loaded by parallel resistance combinations equivalent to 5 kΩ, the gain for V_1 is $1.4 \times 0.428 = 0.6$, and the gain for V_2 is $1.4 \times 0.57 = 0.8$.

Another summation problem is shown in Fig. 2(c).

Let $R_p = 5\ k\Omega$
$\Sigma a = 0.4$
$(1 +\Sigma b) = 1$
$(1 + \Sigma b) - \Sigma a = 0.6$ *(An R_O is needed.)*
$R_F = 1 \times 5\ k\Omega = 5\ k\Omega$ *(Noise gain is 1.)*
$R_O = 5/0.6 = 8.3\ k\Omega$
$R_1 = 5/0.1 \doteq 50\ k\Omega$
$R_2 = 5/0.3 = 16.7\ k\Omega$

The load on the inverting terminal is only R_F, which is 5 kΩ. The load on the noninverting terminal, consisting of the parallel combination of R_O, R_1, and R_2, is also 5 kΩ. The gain for V_1 is the product of noise gain multiplied by attenuation, or $1 \times 5.5/55 = 0.1$. The gain for V_2 is $1 \times 7.1/23.8 = 0.3$.

The last example, which is not as trivial as it looks, is the calculation of resistances for the inverting adder in Fig. 2(d).

Let $R_p = 5\ k\Omega$
$\Sigma a = 0$
$(1 + \Sigma b) = 2.5$
$(1 + \Sigma b) - \Sigma a = 2.5$ *(R_O is needed.)*
$R_F = 2.5 \times 5\ k\Omega = 12.5\ k\Omega$ *(Noise gain is 2.5.)*
$R_O = 12.5/2.5 = 5\ k\Omega$
$R_1 = 12.5/0.3 = 41.7\ k\Omega$
$R_2 = 12.5/1.2 = 10.4\ k\Omega$

A check of these results shows R_1, R_2, and R_F in parallel have a total resistance of 5 kΩ. Gain for V_1 is $-2.5 \times 0.02 = -0.3$, and gain for V_2 is $-2.5 \times 0.48 = -1.2$. □

Engineer's Notebook is a regular feature in Electronics. We invite readers to submit original design shortcuts, calculation aids, measurement and test techniques, and other ideas for saving engineering time or cost. We'll pay $50 for each item published.