Semilog paper is short cut to finding filter frequencies

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Semilogarithmic graph paper provides a handy way to estimate center frequencies and band-edge frequencies in the design of filter banks. It's also a convenient way of finding fractional roots and powers of numbers.

Both applications make use of the fact that a straight line on semilog paper represents the functional relation that can be stated as:

$$\log y = a + bx \qquad (1)$$

or

$$y = y_0 k^x \qquad (2)$$

where y is 10^a and k is 10^b.

For example, an engineer may want to design a filter bank in which the ratio of successive center frequencies ($f_0, f_1, \ldots f_{N-1}$) is constant:

$$f_1 = cf_0$$
$$f_2 = cf_1 = c^2f_0$$
$$f_3 = cf_2 = c^3f_0, \textit{etc.}$$

or, in general,

$$f_n = c^n f_0 \qquad (3)$$

Equation (3) has the same form as Eq. (2), so a semilog graph of the frequency of each filter stage, plotted against the number of that stage, is linear. Therefore the frequency of the first stage can be plotted at abscissa zero, the frequency of the last stage can be plotted at abscissa (N – 1) where N is the total number of stages in the filter, and when the two points are connected by a straight line, the frequencies of all intermediate stages can then be read at a glance.

Thus in Fig. 1 the line A illustrates how to estimate center frequencies given a requirement for seven channels total, with a lowest-channel center at 80 hertz and a highest-channel center at 500 Hz. The line connecting points (0, 80) and (6, 500) shows that the intermediate frequencies are 109, 147, 200, 271, and 368 Hz.

It should be noted that this graphical technique circumvents the need for some fancy calculation. For instance, it is not necessary to compute c, which in this case is:

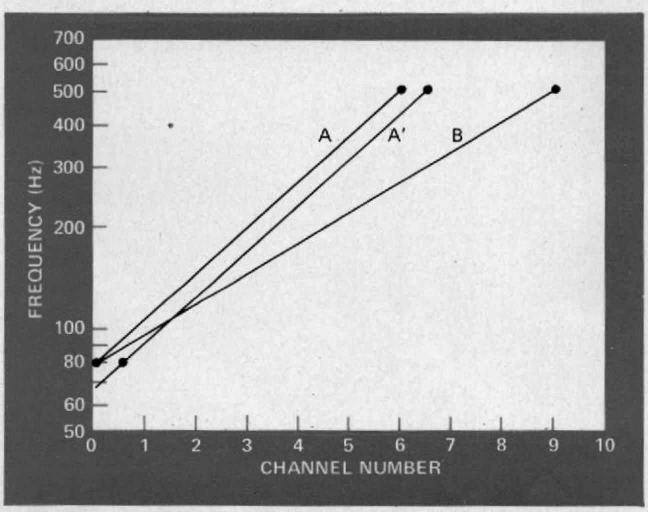
$$c = (500/80)^{1/6} = (5/2)^{1/3}$$

The method lends itself to quick appraisal of alternative filter schemes; for example, to find frequencies for a scheme with 10 channels instead of seven, line B is drawn connecting the point (0, 80) with the point (9, 500). Even if a calculator were at hand, it could not possibly give such a meaningful representation of the desired information in so short a time.

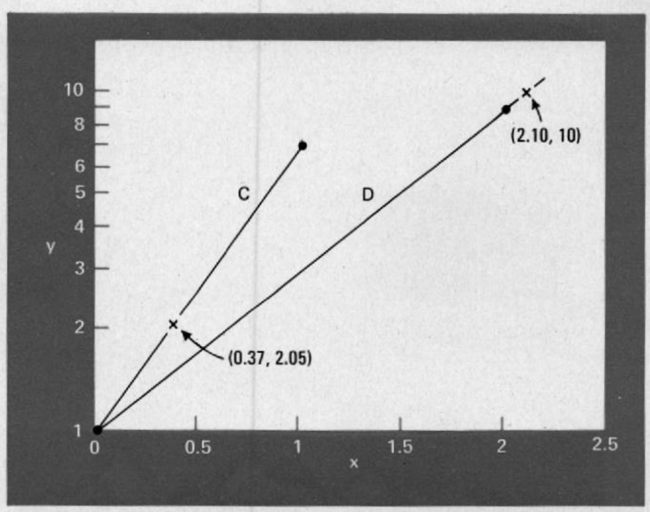
The line that connects the points (½, 80) and (6½, 500), which is labeled A′ in Fig. 1, gives band-edge frequencies that equal the geometric means between successive center frequencies for the seven-stage filter. This sort of information is of interest in the design of constant-Q filters.

Use of semilog paper to estimate fractional roots and powers corresponds to letting y_0 equal unity in Eq. (2). Figure 2 illustrates the technique in finding the value of

1. Filter frequencies. As graphic aid in design of filter banks with constant frequency ratio between stages, line A determines frequencies for a seven-channel system and line B determines them for a 10-channel system, both covering the range from 80 hertz to 500 Hz. Line A′ determines band-edge frequencies for the seven-channel system. The error in reading frequency values is about 0.5%.



2. Roots and powers. Lines C and D illustrate use of semilog graph paper to provide quick solutions for values y and x in equations $y = 7^{0.37}$ and $10 = 3^x$, respectively.



$7^{0.37}$. If:

$$y = 7^x$$

some known relations are:

$$1 = 7^0$$

and

$$7 = 7^1$$

Therefore $y = 7^{0.37}$ is found by drawing a straight line connecting (0, 1) and (1, 7) on the semilog paper. Where x is 0.37, y is found to have the value of 2.05.

As a final example, to find x in the equation $10 = 3^x$ (i.e., to find $\log_3 10$), draw line D to connect the points (0, 1) and (2, 9), and extend it out to y = 10. At y = 10, x = 2.10.

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