CIRCUIT SURGERY

REGULAR CLINIC

BY IAN BELL

High-frequency signals and low-pass filters

A LITTLE while ago user ***741*** posted a question about low-pass filters on the *EPE Chat Zone*. As regular readers will know, this forum has since been retired and *EPE* 'chat' is now hosted on EEWeb (**www.eeweb.com/forum**). The old forum is still available in read-only-mode, but new discussions can be started on EEWeb using the tag 'EPE Magazine' to mark threads as being *EPE*-related.

Returning to ***741's*** question, he asks: 'Op amps have GBW (gain-bandwidth product), say 10MHz. If this was an amplifier circuit (not a filter), then (ignoring phase gremlins), the relation $G = 1/\beta$ gets less accurate as A (open-loop gain) falls with frequency. At 100kHz, A is 100, so the relation holds well, but at 1MHz, we need to use $G = A/(1+A\beta)$... I think.

How does the performance of say an MFB (multiple-feedback) low-pass filter suffer with rising input frequency? Suppose we have a 1kHz low-pass filter. You feed in 10kHz, and it maybe works 'well'. You apply 1MHz, and it starts to act up.'

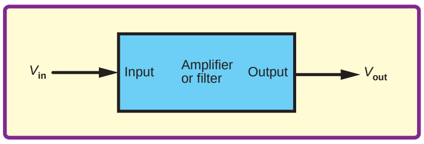

Fig.1. Simple electronic system; the gain is v_{out}/v_{in}.

Back to basics

Starting with circuit fundamentals, a filter or amplifier is an electronic system (see Fig.1) which reproduces the signal on its input (v_{in}) on its output (v_{out}). Typically, the output has different amplitude and power delivery capabilities. The ratio of output to input signal level v_{out}/v_{in} is the voltage gain (G) and very often (as in ***741's*** question) we are interested in how the gain of a circuit varies with frequency, known as the 'frequency response' of the circuit. Often, a frequency response is shown by plotting a graph of gain against frequency (see Fig.2, which we will discuss in more detail shortly). Such a plot can obtained by measurement in a lab, by simulation or by plotting a mathematical function derived from analysis of the circuit. The frequency response graph can be used to help decide if a circuit is appropriate for our intended purpose. Frequency response requirements may be used as part of the specification for a circuit design.

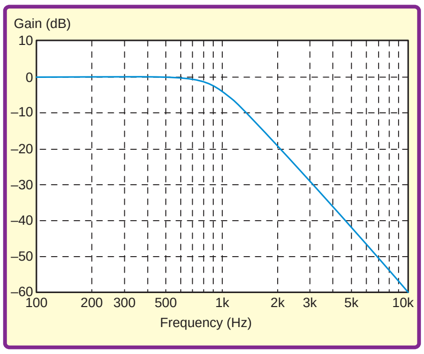

Fig.2. Example frequency response graph of a 1.0kHz low-pass filter.

The question also prompts some more specific points – how we interpret the well-known negative feedback gain equation $G = A/(1+A\beta)$ in the context of op amp filters and why the performance of specific low-pass filter circuits may degrade at relatively high frequencies. We will look at this after introducing some basics of frequency response plots and filter characteristics.

Filters are circuits that pass signals (from input to output) for a certain range of frequencies (called the pass band) while rejecting signals at other frequencies (in the stop band). For signals within the pass band the filter may have high gain, low gain or even attenuation – the key thing is not the specific gain, but the fact that the gain will diminish significantly with respect to the pass band for frequencies outside this pass band. As its name implies, a low-pass filter should pass signals only if they are below a given frequency, called the 'cut-off frequency'.

Decibels

On frequency responses graphs gain is often plotted using a scale in decibels (dB) versus the logarithm of frequency. The definition of a decibel is also logarithmic and is based on the power ratio of two signals P_1 and P_2 – specifically, the power ratio in decibels is given by: $10 \times \log_{10}(P_2/P_1)$ dB, where P_2/P_1 is the power gain (eg, P_{out}/P_{in}), or P_1 is an agreed reference level and P_2 is a measured value. The term 'decibel' means one tenth (deci, hence d) of a bel (symbol B). One bel is $\log_{10}(P_2/P_1)$, but as we use $10 \times \log_{10}(P_2/P_1)$ we are counting in tenths of a bel. The bel is named after Alexander Graham Bell.

The definition of the decibel is based on power, but we are often interested in voltage levels and voltage gains. Power is related to the square of voltage, specifically $P = V^2/R$ for a voltage (V) across a resistor (R). If we square something inside a logarithm it is equivalent to multiplying the log by two (without the square); that is, $\log(y^2) = 2 \times \log(y)$. So for voltage gains, we assume a reference resistor (R) that cancels out when we find $P_{out}/P_{in} = (V_{out}{}^2/R)/(V_{in}{}^2/R) = V_{out}{}^2/V_{in}{}^2$, we get a voltage gain in decibels of $2 \times 10 \times \log_{10}(V_{out}/V_{in}) = 20 \times \log_{10}(V_{out}/V_{in})$.

On the decibel scale, a gain of 1 is 0 dB, gains greater than 1 (amplification) are positive and gains less than one (attenuation) are negative. The graph in Fig.2 shows a filter with unity-gain at low frequencies and increasing attenuation at high frequencies. The scale points of –10, –20, –20, –40… correspond to attenuations of 1/3.2, 1/10, 1/32 and 1/100 respectively. To covert from a voltage gain (G) expressed in dB to a simple numerical gain find $10^{(G/20)}$. For a power gain use $10^{(G/10)}$.

Being able to interpret decibel values, and plots using decibels and log frequency is very useful when working with filters. The log-log scale allows details of responses to be seen over a very wide range of frequency and gain, which would be lost with a linear plot. However, if they are not read correctly the importance of small features may seem exaggerated – you need to be able to interpret the dB values in terms of their actual relevance to the circuit you are working with. Having a feel for what the numbers mean (eg, a –40dB output is 100 times smaller than the input voltage) helps.

Brick wall

An ideal low-pass filter would pass all signals below the cut-off and completely reject all signals above the cut-off. The frequency response graph would look like Fig.3. Unfortunately, this perfectly sharp cut-off, which is referred to as a 'brick wall' filter, cannot be physically implemented.

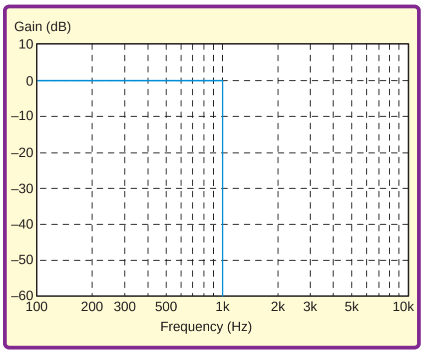

Fig.3. Frequency response graph of a 1.0kHz 'Brick Wall' low-pass filter. This perfect filter cannot be built as a real circuit.

Instead, real filters have a more gradual decrease in gain in the transition from pass band to stop band; for example, the frequency response in Fig.2. The question then arises as to where the cut-off is taken to be along the smooth curve. The usual answer is that the cut-off is defined to be the frequency at which the output power from the filter falls to half that in the pass band, this is $10 \times \log_{10}(0.5) = -3$ dB. For example, for the response shown in Fig.2, the cut-off frequency is 1kHz (the same as the brick wall filter in Fig.3).

All filters deviate from the ideal brick wall filter, so the question a circuit designer faces is what is good enough, or most suitable for the application? The purpose of a filter is to remove unwanted frequency content from a signal because it will cause problems in the following circuitry or output signal. The attenuation of the unwanted signal has to be sufficient to prevent its presence from being problematical. The filter specification becomes more demanding if the wanted and unwanted signals are close in frequency, because this requires a filter to be closer to the ideal, but impossible-to-achieve brick wall response. In many situations where a filter is required the unwanted frequencies will be close, or moderately close, to the wanted signal, and there may be very little unwanted signal strength at much higher frequencies. In such cases the performance of the filter at these higher frequencies may not be particularly important. However, this is not always the case and some applications demand filters with good performance over a very wide range of stop band frequencies. It is therefore important to understand the limitations of some types of low-pass filter circuit at high frequencies. This is exactly the issue raised by ***741***.

Feedback equation

741's question makes reference to the feedback gain equation $G = A/(1+A\beta)$, in which G is the gain of the whole circuit with feedback, A is the gain of the amplifier to which feedback is applied and β is the proportion of the output signal applied as negative feedback. A is called the 'open-loop gain' of the amplifier, this does not change when the external feedback is applied – it is an internal property of the amplifier. However, the gain of the whole circuit is strongly dependent on the feedback, as well as being influenced by A. We have discussed feedback theory in *Circuit Surgery* (November 2016) and showed how this equation simplifies to $G = 1/\beta$ if A is very large (eg, hundreds of thousands), which it typically is for op amps at low frequencies. This is an important result because it means the gain of the circuit only depends on the external feedback components (eg, the resistors in an op amp amplifier circuit) and not on the value of the open-loop gain of the amplifier itself.

The $G = A/(1+A\beta)$ equation is obtained by drawing a system diagram to represent the relationships between the signals in the circuit and finding the gain (output/input) of the system. This diagram is an abstract representation of the behaviour of the circuit and is used because it provides a useful basis for mathematical analysis. System diagrams like this are frequently used in control systems design – the technique is applicable well beyond analysis of circuits such as amplifiers.

System diagrams

Some standard op amp amplifier circuits are shown in Fig.4. The most straightforward case for developing a system diagram is the non-inverting amplifier. Here it is clear to see that the output is fed back to the inverting input via a potential divider. The fraction of the output voltage that appears at the junction of the two resistors is the feedback fraction β. The circuit input is applied directly to the non-inverting input of the op amp. Thus the input is added and the feedback subtracted at the amplifier input. This leads to the system diagram in Fig.5, which can be used to derive the equation ***741*** quotes. This was done in the November 2016 article, so we will not repeat it here.

An important point about the system diagram in Fig.5, and the equation obtained from it, is that it applies specifically to the non-inverting amplifier. It may, but does not necessarily, apply to other amplifier circuits. For example, as was also pointed out in the November 2016 article, the system diagram for the inverting amplifier is not exactly the same as the one in Fig.5. This means that the equations $G = A/(1+A\beta)$ and $G = 1/\beta$ do not apply to the inverting amplifier; for example the simplified equation for the inverting amplifier is $G = (1–1/\beta)$. This is not always made clear when op amp feedback theory is discussed.

The non-inverting amplifier is sufficient for explaining the general principles of feedback theory, so the variants are not presented, and it is easy to assume the non-inverting equations apply to all circuits. This is directly relevant to ***741's*** question – in a later post he commented 'It's hard to work out what β is for many filters'. Filter circuits may have more complex feedback structures than basic amplifiers, so the assumption that the non-inverting amplifier feedback equation can be straightforwardly applied may be wrong.

To obtain equations for the input-output relationships of single op amp filters, like the MFB filter ***741*** mentions, we typically apply basic circuit theory (eg, Ohm's and Kirchhoff's laws) together with idealised op amp characteristics (zero input current, and zero voltage across the inputs due to the very high gain in combination with negative feedback). On the other hand, the basic theory that gives us $G = A/(1+A\beta)$ and $G = 1/\beta$ does help explain, in general terms, what happens to op amp circuits as frequency increases and this is related to high-frequency performance issues with filters such as the MFB.

Compensation

Fig.6 shows the frequency response of a typical op amp's open-loop gain and the closed-loop gain of an amplifier (eg, inverting and non-inverting amplifiers) built using the op amp. In

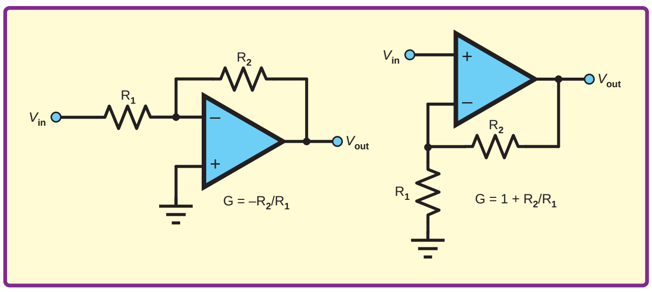

Fig.4. Op amp amplifiers

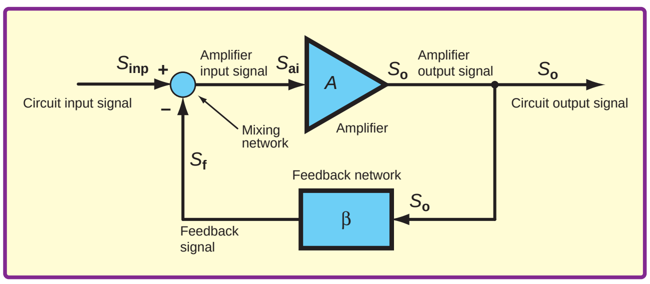

Fig.5. System diagram for the non-inverting op amp amplifier

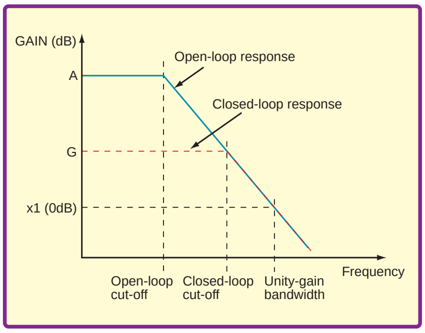

Fig.6. Typical form of the open- and closed-loop frequency responses of an op amp amplifier.

both cases this is a low-pass response. The direct-coupled, DC-amplifying, nature of the op amp circuit means that the gain does not also roll off at low frequencies, as it would for a capacitively coupled amplifier. The open-loop op amp has full gain at DC, but this starts dropping at a very low frequency compared to those at which many op amp applications operate. The cut-off frequency is typically 1 to 10Hz, with the falloff in gain above that frequency being 20dB per decade (tenfold increase in frequency). This low cut-off frequency for the open-loop gain is deliberately designed into the op amp to prevent instability when feedback is applied and is known as 'compensation'.

The closed-loop gain (G) of an op amp amplifier is set by the feedback resistors, as mentioned above, and is usually much lower than the open-loop gain. As frequency increases, the closed-loop gain remains constant at $1/\beta$ until the approximation $A = 1/\beta$ no longer holds due to the open-loop gain (A) at that frequency (and above) being too low. At these higher frequencies the closed-loop gain decreases along with the open-loop gain (see Fig.6). The cut-off frequency for the closed-loop response is typically at a much higher frequency than the cut-off frequency point of the open-loop gain.

As the op amp's gain extends from DC, the frequency at which the closed-loop gain starts dropping off is equal to the circuit's bandwidth (range of frequency it can amplify) at that closed-loop gain. The lower the closed-loop gain, the higher the frequency at which the closed-loop frequency response intersects the open-loop response and starts decreasing. The straight-line nature of the open-loop response in Fig.6 means that if you multiply the closed-loop gain and closed-loop bandwidth together for any closed-loop gain you get the same value. This is called the 'gain-bandwidth product' (GBW) and is an indication of how 'high frequency' an op amp is. Fixed GBW is common for op amps, but not universal, as it depends on the form of compensation used.

From the preceding discussion we see that application of negative feedback to a very high gain amplifier (op amp) makes the circuit gain insensitive to the gain of the amplifier itself. This effect breaks down at frequencies where the amplifier's gain is not significantly larger than the circuit gain. Negative feedback has other significant effects on circuit performance, for example it reduces the effective output impedance. Like amplifier-independent gain-setting, the feedback magic is lost for these effects too at high frequencies; for example, effective output impedance may increase at high frequencies. The equations $G = A/(1+A\beta)$ and $G = 1/\beta$ quoted by ***741*** may only relate to gain and may only apply to specific circuits but, together with the op amp frequency response curve, they demonstrate the general principle of feedback providing improved circuit performance, which is lost at high frequencies due to diminished open-loop gain. The problem with the MFB's (and similar low-pass filters') gain increasing at high frequencies is due to an increase in output impedance.

Analysis

Fig.7 shows a Sallen-Key filter, which is a single op amp filter circuit, like the MFB filter mentioned by ***741***. At very high frequencies the capacitors in the circuit behave like short circuits leading to the simplified version of the circuit shown in Fig.8. The op amp has been replaced by a unity-gain buffer (this is the role it performs in this circuit). Given that the buffer's input is shorted to ground, the buffer's internal voltage source is producing 0V. This is the same as connecting the internal side of the output impedance to ground, allowing us to further simplify the equivalent circuit to that shown in Fig.9.

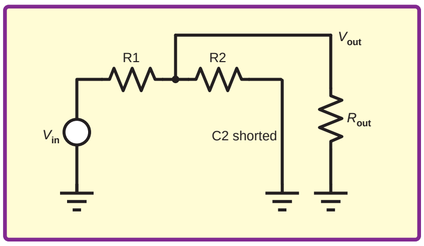

Fig.9. Further simplification of the equivalent circuit in Fig.8.

Looking at Fig.8 we see that R_2 and R_O are in parallel, but typically R_2 is at least 100-times larger than R_O, so we can ignore R_2. Thus v_{out} is related to v_{in} by the potential divider formed by R_1 and R_O, and so:.

$$V_{out} = \frac{R_O v_{in}}{(R_O + R_1)}$$

However, again, R_1 is typically about 100-times larger than R_O, so in this case we can ignore R_O in the sum of the resistances, giving an approximate value of v_{out} as:

$$V_{out} = \frac{R_O}{R_1} v_{in}$$

The output of the real filter circuit is a combination of the unwanted signal given by the above equation and the ideal filter response. At low frequencies the ideal response dominates, but as frequency increases, the op amp's gain decreases, so the effective value of R_O increases, increasing the contribution of the unwanted signal to the total output. At high frequencies this unwanted signal dominates the circuit's output. The MFB suffers in a similar way, although the equivalent circuit model is different.

Simulation

The behaviour of Sallen-Key and MFB filters at high frequencies can be observed using simulations. Fig.10 shows a schematic used for a simulation of MFB and Sallen-Key filters implemented using Linear Technology's model of their LT1002A op amp, which is supplied with LTspice. The Sallen-Key filter is also implemented with an ideal op amp

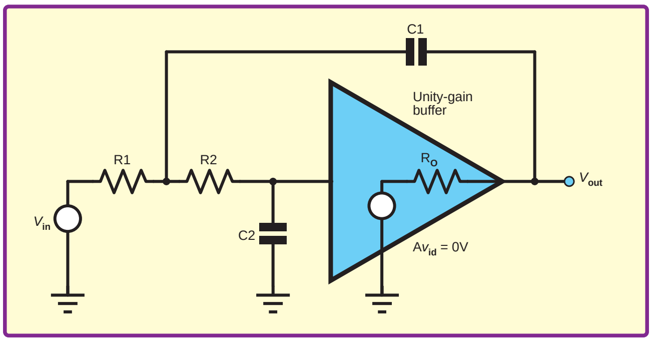

Fig.7. Sallen-Key filter.

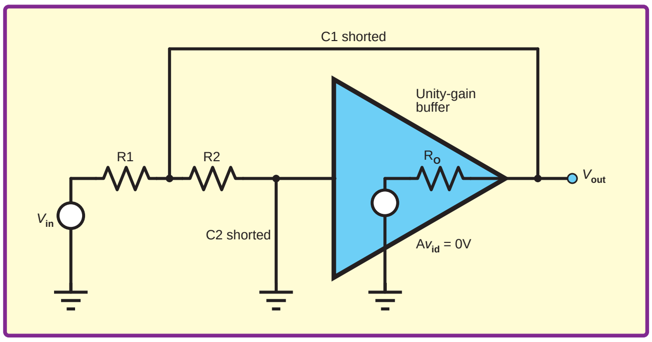

Fig.8. Simplified equivalent circuit for the filter in Fig.7 at very high frequencies.

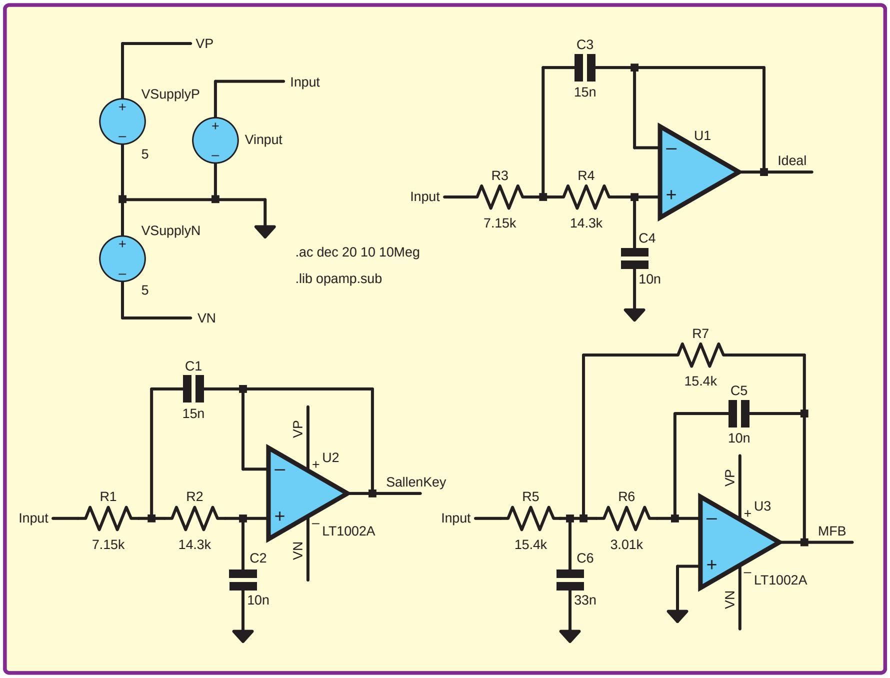

Fig.10. Schematic for LTspice simulation comparing ideal, Sallen-Key and MFB low-pass filters.

model, which will have zero output impedance at all frequencies and hence not suffer from the problem described above.

The results of the simulation are shown in Fig.11 and demonstrate the issue mentioned by ***741***. It can be seen that both the Sallen-Key and MFB filters suffer from the problem of increased gain at high frequencies in comparison with the ideal circuit, but the MFB circuit exhibits better performance. The exact performance of the circuits will depend on the op amp used and relevant external component values. The simulation only shows the effect of using the realistic op amp model. The influence of other components' non-ideal behaviour and physical layout are not included, but may need to be considered in real circuits at high frequencies. In this example, the Sallen-Key circuit attenuates signals above 1MHz by 40dB (one hundredth), whether or not this is acceptable would depend on the application. The MFB circuit is much better: –100dB at 1MHz is an attenuation of one hundred-thousandth.

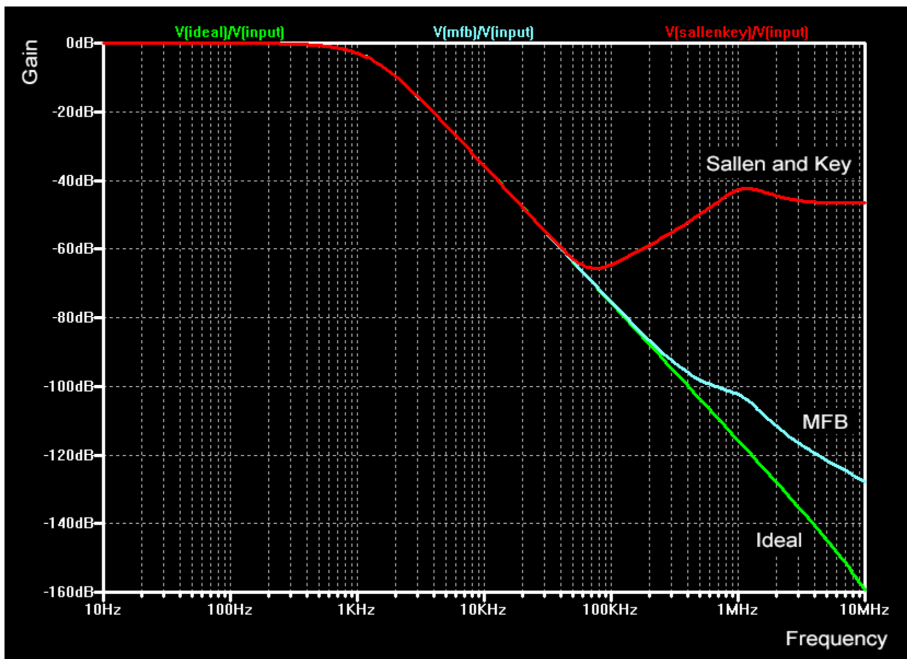

Fig.11. LTspice simulation results from the circuit in Fig.10.

Further reading

Readers interested in more details on this topic are directed towards Jim Karki's application reports from Texas Instruments.

Karki, J., Active Low-Pass Filter Design, Rev. B, 2002, *Texas Instruments Application Report SLOA049B*.
www.ti.com/litv/pdf/sloa049b

Karki, J., Analysis of the Sallen-Key Architecture, Rev. B, 2002, *Texas Instruments Application Report SLOA024B*.
www.ti.com/litv/pdf/sloa024b