Designing Active Filters

Part 2 (Conclusion)

A short-cut method for practical design of first-through-third-order active filters

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Last month in Part 1 of this article, we introduced you to active-filter basics and discussed how to design first-order filters. In this conclusion, we focus on second- and third-order filters.

Second-Order Filters

It should be noted that all first-order filters are Butterworth types. Second-order and beyond filters, however, allow you to choose the type of filter. You might be surprised to learn that second-order filters require only one op amp. What makes them different from first-order filters is that they have an extra resistor and capacitor. Like all second-order networks, the ultimate slope is 40 dB/decade.

The amount of "damping" a filter has determines whether it is a Butterworth, Bessel or Chebychev design. Damping is the resistive loss built into the filter to keep it under control. Critically damping a filter gives it a Butterworth characteristic, which has the flattest possible bandpass and exhibits complete freedom from overshoot. Underdamping yields the Chebychev filter, which is overly "bouncy." Overdamping gives the Bessel filter, which has a sag in the passband before cutoff.

Damping in filters is controlled by the ratio used in calculating certain component values. You don't have to compute the damping ratio. This has already been done for you by others; all you have to do is use their numbers. The damping figures shown in the Tables are from Don Lancaster's *Active Filter Cookbook* (Howard W. Sams).

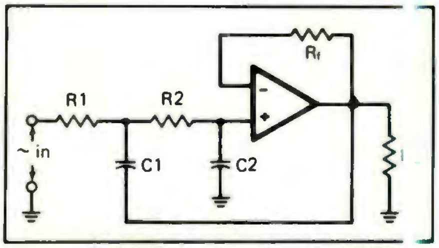

Fig. 3. A second-order, low-pass unity-gain active filter.

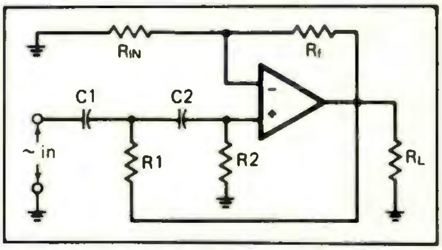

Fig. 4. A second-order, high pass unity-gain active filter.

• *Low-Pass Filter.* For the present, let's concentrate on the Butterworth filter, whose damping ratio is 1.414. Component values affected by this ratio depend on what kind of circuit you're using. A low-pass unity-gain filter circuit is shown in Fig. 3. Note in this arrangement that *C2* goes to ground, while *C1* provides a path for some of the op amp's output to be fed back to the input of the filter network made up of *R2* and *C2*. This arrangement, not possible in a passive filter, allows you to do away with the inductor in an active filter.

With the Fig. 3 circuit, the damping ratio is used to determine the values of *C1* and *C2*. We'll still compute (or scale from the reference filter), but this time the result will be only the starting point—not the final value—for each capacitor. In essence, what you'll get are the "average" values for *C1* and *C2*. The *actual* values of these capacitors are calculated using the average values and the damping ratio figure.

For second-order low-pass filters only, C1 = C(2/d) and C2 = C(d/2). Here *d* is the damping ratio and *C* is the average value of the capacitor. Also, R_f = R1 + R2.

Start with the design of a second-order low-pass Butterworth filter in which F_C = 1 kHz and the values of both *R1* and *R2* are 10,000 ohms. Working from the reference filter discussed above, you know that the average values of *C1* and *C2* are both 0.16 μF. Using the actual-value formulas: C1 = 0.016(2/1.414) = 0.0226 μF; C2 = 0.016(1.414/2) = 0.0113 μF; and R_f = 10,000 + 10,000 = 20,000 (20k) ohms.

Note that C1 = 2C2. It will always be this way for a unity-gain, second-order, low-pass Butterworth filter. If you're analyzing a circuit that has already been designed, you can get the average value of *C* by taking the geometric average of the two capacitors in the circuit: C = $\sqrt{0.0226 \times 0.0113}$ = 0.016 μF. Then calculate the cutoff frequency: F_C = 1/(6.28 × 10,000 × 0.016) = 995 Hz. Note that because the average value of *C1* and *C2* (not either one alone) determines F_C, the 0.016-μF value is used in the last equation.

• *High-Pass Filter.* There are only

Table 1. Second-Order Filter Factor			
Filter Type	**Frequency Correction Factor High-Pass**	**Low-Pass**	**Damping**
Bessel	0.785f	1.274f	1.732
Butterworth	1.000f	1.000f	1.414
Chebychev			
1 dB	1.159f	0.863f	1.045
2 dB	1.174f	0.852f	0.895
3 dB	1.189f	0.841f	0.767

*Data taken from *Active Filter Cookbook* by Don Lancaster (Howard W. Sams & Co., Inc.)

four things that make the second-order, high-pass, unity-gain filter shown in Fig. 4 different from the low-pass configuration shown in Fig. 3. Firstly, the positions of the capacitors and resistors are reversed. Secondly, you don't calculate special *C1* and *C2* values (the value of *C* obtained by scaling from the reference filter is used as is for both capacitors). Thirdly, you treat the scaled value of *R*, from the reference filter, as an "average" value, which is used along with the damping ratio to compute the final values from: R1 = R(d/2) and R2 = R(2/d). Finally, feedback resistor R_f is the average value of *R*.

Design a high-pass, unity-gain Butterworth filter in which F_c = 1 kHz and *R* = 10,000 ohms, which means that *C* must be 0.016 μF. Therefore, R1 = 10,000 × (1.414/2), R2 = 10,000 × (2/1.414), R_f = R = 10,000 ohms and C1 = C2 = 0.016 μF.

• *Chebychev Filters.* In this category is a whole family of filters. With the Chebychev filter's steep rolloff close to cutoff comes ripple throughout the passband and poorer phase-shift performance. Each member of the Chebychev family has a different combination of rolloff slope steepness versus passband ripple amplitude. The steeper the rolloff, the more ripple you have to accept.

One good thing about Chebychev filters is that you get to decide what you want. If you need a modest improvement in slope steepness, you can get it with very little ripple. On the other hand, if you want a greater increase in slope but don't mind a lot of ripple, you can get that, too.

Ripple is measured in decibels. Chebychev filters are classified by the amount of ripple they have. Thus, a 1-dB Chebychev filter has a bump that rises 1 dB above the passband; the bump in a 2-dB deisgn rises by that amount; and so on.

With the Chebychev filter, you're dealing with at least a second-order circuit. Designing the filter is simply a matter of adjusting frequency and damping to suit your needs. The formulas don't change; they're the same as those used above. The only difference is the values used for frequency and damping in calculations.

Unless you're making a Butterworth filter, the value of *F* you plug in isn't the same as F_c. You must multiply the desired F_c by the correction factor shown in Table 1. For example, if you want F_c to be 1 kHz in a second-order, low-pass 3-dB Chebychev filter, you first determine the correction factor, which is 0.841 in Table 1. Then multiply the 1-kHz F_c by the correction factor. This gives the cutoff frequency of the finished filter, which will be 1 kHz.

Note in Table 2 that each response shape has its own damping ratio. For the Butterworth filter, the damping factor is 1.414. Whenever a formula calls for a value of *d*, you'd use the 0.767 figure instead of 1.414. The high- or low-pass filter circuit itself doesn't change; it's the same as for the Butterworth filter.

Now design a low-pass, second-order 3-dB Chebychev filter in which F_c = 1 kHz and *R* = 10,000 ohms. From Table 1, F = 0.841(F_c) = 0.841 × 1000 = 841 Hz and d = 0.767. Then $C_{average}$ = 1/(6.28 × FR) = 1/(6.28 × 841 × 10,000) = 0.018 μF; C1 = 0.0189(2/0.767) = 0.0493 μF; C2 = 0.0189(0.767/2) = 0.00725 μF; R1 = R2 = 10,000 (10k) ohms; and R_f = R1 + R2 = 20,000 (20k) ohms.

Now to design a high-pass, second-order 2-dB Chebychev filter where F_c = 1 kHz. Obtain the *R* and *C* values from the reference filter. Then F = 1.174(F_c) = 1174 Hz; C1 = C2 = 1/(6.28 × 1174 × 10,000) = 0.0316 μF; R1 = 10,000 × (0.895/2) = 4475 ohms; R2 = 10,000 × (2/0.895) = 22,350 ohms; and R_f = 10,000 ohms.

Table 2. Third-Order Frequency Factors & Damping Ratios*			
Section Type	**Frequency High-Pass****	**Factors Low-Pass****	**Damping Ratio (Second-Order)**
Bessel	0.753/0.688	1.328/1.454	1.447
Butterworth	1.000/1.000	1.000/1.000	1.000
Chebychev			
1 dB	2.212/1.098	0.452/0.911	0.496
2 dB	3.105/1.095	0.322/0.913	0.402
3 dB	3.344/1.092	0.299/0.916	0.326

*Data taken from *Active Filter Cookbook* by Don Lancaster (Howard W. Sams & Co., Inc.)
**The first figure in these two columns is for the first-order filter, the second for the second-order filter.

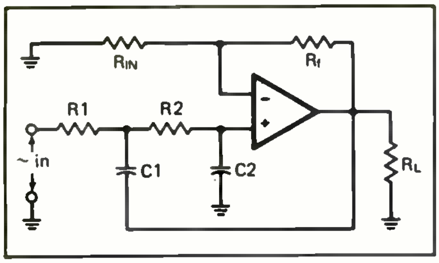

Fig. 5. A second-order, low-pass equal-component-values active filter.

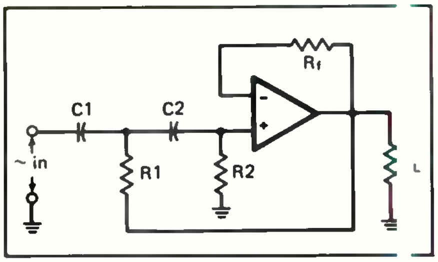

Fig. 6. A second-order, high-pass active filter.

You may have noticed that with second-order filters the unequal values of *C1* and *C2* may be difficult to find. The low- and high-pass filter ciruits shown in Figs. 5 and 6, respectively, simplify matters by allowing you to use a single value for both *C1* and *C2*. Similarly, one value of resistance does the job for both *R1* and *R2*. An important new feature of these equal-component-value circuits is that input resistor R_{in} has been added. The value of R_{in} is 39,000 (39k) ohms.

In the Fig. 5 and 6 circuits, damping is controlled by the gain of the op amp. Since the noninverting (+) input is being used, gain is (R_f/R_{in}) + 1. The gain needed, in turn, depends on the damping required. It's always calculated as 3 − d.

If R_f = 23,000 ohms, gain is (23,000/39,000) + 1 = about 1.59. If 1.59 = 3 − d, *d* is about 1.414, which is the required damping for a Butterworth filter. Thus, you can make a second-order, low-pass Butterworth filter in which F_c = 1 kHz by making R_f = 23,000 ohms and using the normalized values given above.

The catch is that the filter's pass-band gain will no longer be unity. It will now be 3 − d. Hence, in our last example, gain is 1.59. Since each response shape's damping value is different, each filter type must have its own unique gain. Normally, this isn't a disadvantage, since a gain of unity is seldom needed.

Using the equal-component-value circuit, design a low-pass, second-order 2-dB Chebychev filter in which F_c = 1.5 kHz, R1 = R2 = 10,000 ohms, C = 0.016 μF and R_{in} = 39,000 ohms. Given these parameters, C = 0.016 × (1000/1500) = 0.016 μF; d = 0.895 (from Table 2); and Gain = 3 − 0.895 = 2.105. Since Gain = (R_f/R_{in}) + 1, plug in the known values and you have 2.105 = (R_f/39,000) + 1. Rearranging the formula gives: R_f = (2.105 − 1) × 39,000 = 43,100 (43.1k) ohms.

Since the correction factor of 0.852 from Table 2 indicates adjustment to a lower frequency, you must raise the value of *C* or *R*. Adjusting *R* yields R1 = R2 = 10,000/0.852 = 11,740 ohms. The final calculated values now become: C1 = C2 = 0.01 μF; R1 = R2 = 11,740 ohms; and R_f = 43,100 ohms.

Following the above steps is all it takes to design any second-order filter when you use the equal-component-value circuit.

Third-Order Filters

Once you know how to design first- and second-order filters, designing higher-order filters is a cinch. To make a third-order filter, you just follow a first-order filter with a second-order section. The latter can be either a unity-gain or equal-component-value design. The only added complication is that you must now use a new set of frequency-correction and damping values for both filter sections. Table 2 lists the proper values to use for third-order filters.

Armed with the above information, design a third-order, high-pass 2-dB Chebychev filter in which F_c = 800 Hz. Use the equal-component-value circuit for the second-order section, and design by scaling from the reference circuit's values.

For the first-order section, the frequency factor (from Table 2) is 3.105. Hence, this section must be designed for 3.105 × 800 Hz = 2848 Hz. Assuming C = 0.016 μF and R = 10,000 ohms, rescale: R = (10,000 × 1000)/2484 = 4026 ohms.

For the second-order section, the frequency factor (again from Table 2) is 1.095. Therefore, this section must be designed for 1.095 × 800 = 876 Hz. Now assuming C = 0.016 μF and R = 10,000 ohms, scaling tells you that R = (10,000 × 1000)/876 = 11,400 ohms. Table 2 also tells you that d = 0.402. Hence, Gain = 3 − 0.402 = 2.6, which gives you 2.6 = (R_f/39,000) + 1, giving you R_f = 62,400 ohms.

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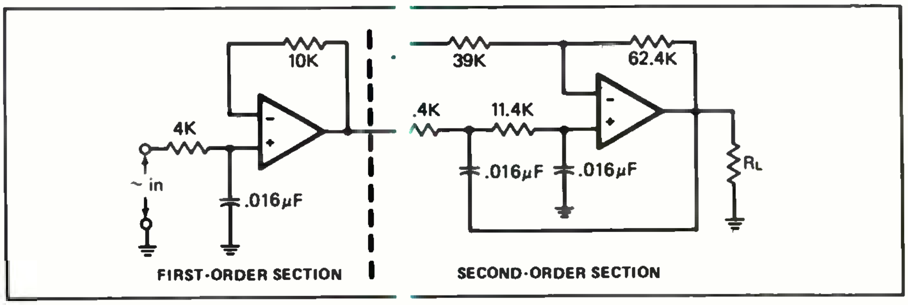

Fig. 7. A third-order, low-pass active filter consists of separate first- and second-order sections. Each is designed independently of the other.

Active Filters *(from page 58)*

The schematic diagram of the circuit just designed, with component values indicated, is shown in Fig. 7.

More Information

In this article, we've discussed practical design approaches to first-, second- and third-order Bessel, Butterworth and Chebychev active filters on a more or less elementary level. Topics not covered include: filters beyond the third-order; filters with response shapes between those discussed; bandpass filters; filters with variable cutoff frequencies; and use of filters as crossover networks in audio systems. If you wish to learn about these and much more, there are a number of books to which you can refer. Two good ones are Don Lancaster's *Active-Filter Cookbook* and W.G. Jung's *Audio Op-Amp Applications*, both published by Howard W. Sams. ME